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Growth and Residential Distribution with Solow, Alonso, and Dixit-Stiglitz Integrated

Abstract *The purpose of this study is to deal with dynamic interdependence between economic growth, economic structure, and residential distribution. It develops a spatial dynamic economic model on basis of microeconomic foundation. It integrates the economic mechanisms of the Solow one-sector growth model, the Alonso spatial residential model, and the Dixit-Stiglitz equilibrium model with imperfect market. We apply neoclassical economic growth of perfect competition to describe the growth determinant, the neoclassical urban residential model to determine residential location, and the basic model of new growth theory with imperfect market to take account of perfect and imperfect competition in spatial equilibrium structure. The basic economic mechanisms of the three approaches are integrated by using Zhang's new approach to formally model household behavior. We determine the motion by simulation. Then we conduct comparative dynamic analysis to analyze how exogenous changes in different parameters affect residential distribution, economic growth, and economic structure. The study shows how changes in preferences and technologies affect economic growth, economic structure, land rent, and residential distribution.*

Key Words: Urban Dynamics, Alonso's Urban Model, Solow's Growth Model, Dixit-Stiglitz Model; Imperfect Competition, Capital Accumulation, Residential Distribution, Land Rent

Introduction

The purpose of this study is to deal with dynamic interdependence between economic growth, economic structure, and residential distribution. It addresses some economic questions which are not analyzed within an integrated framework. For instance, one can ask what happens to growth, housing choice (measuring in lot size and distance to the city center), economic structure (distribution of labor and capital between sectors and output level of each sector) if the propensity to save is reduced or the propensity to consume one good is shifted. One may also ask similar questions about technological changes and land resource. This paper develops a spatial dynamic economic model on basis of microeconomic foundation. It is built by integrating the ideas in three main models in the literature of theoretical economics - the Solow one-sector growth model (Solow, 1956), the Dixit-Stiglitz equilibrium model with imperfect market (Dixit and Stiglitz, 1977), and the Alonso urban residential model (Alonso, 1964). We apply the mechanism of neoclassical economic growth with perfect competition to describe the growth determinant, the neoclassical urban residential model to determine residential location, and the basic model of new growth theory with imperfect market to take account of perfect and imperfect competition in spatial equilibrium structure.

Any economic event takes place in time and space. A proper economic analysis should not deal with the two dimensions of an event in separated frameworks like in most of theoretical models in economics. Time has always a central concern of theoretical economics. But space has caused great attention in the literature of theoretical economics, especially after Krugman published a few important papers on spatial economics (e.g., Krugman, 1979, 1980, 1998; Zhang, 1991, 1993; Henderson and Thisse, 2004; and Picard and Zenou, 2018). New economic geography provides many new insights into the role of space in economic geography (see also, e.g., Lancaster, 1980; Waterson, 1984; Grossman and Helpman,1990; Benassy, 1996; Bertoletti and Etro, 2015; and Parenti, *et.al.*, 2017). The core model in new economic geography is due to the modelling framework proposed by Dixit and Stiglitz (1977). Krugman applies the approach to economic geography. The new modelling direction provides an analytical framework which

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enables economists to model spatial economies with perfect and monopolistic competitive markets. This modelling technique makes it more realistic to analyze macroeconomic and spatial economic issues than most of traditional economic geographic models which study either perfectly competitive markets or imperfect markets but not with both. Zhang (2018) contributes the literature of new economic growth theory by integrating monopolistic competition with neoclassical growth theory within the Dixit-Stiglitz modelling framework for imperfect competition. This study contributes to the literature by integrating the Solow one-sector growth model and Dixit-Stiglitz model with the Alonso model. This will open an alternative way to analyzing economic geographical issues with a modelling framework of neoclassical growth theory, neoclassical urban economics, new economic geography, new growth theory synthesized. It should be noted that this study provides a way rather than a comprehensive synthesis because there are many models in each of the four theories just mentioned.

As mentioned in Zhang (2018), neoclassical growth theory is developed with perfect competition economies with capital accumulation. The early literature is reviewed by Burmeister and Dobell (1970; see also Richardson, 1973; Henderson, 1985; Henderson and Thisse, 2004; and Zhang, 2005), while new growth theory mainly deals with perfect and imperfect competition without physical capital accumulation. Most of recent developments in new economic geography have neglected wealth accumulation. A main reason for this omission is due to the lack of proper analytical framework to include physical capital and wealth accumulation with microeconomic foundation, except a modelling framework developed by Zhang in 30 years ago which was designed to include endogenous wealth accumulation with time and space as applied in this study. The importance of including wealth accumulation in spatial economics is emphasized by Arnott (1980: 53) in the following way: "In the last decade the static theory of residential urban location and land use has been extensively developed. The theory has generated many useful insights, but because it ignores growth and durability of housing and urban infrastructures there are many urban phenomena it cannot explain." The situation is improved only slightly as far as spatial dynamics with microeconomics is concerned in the literature of mainstream economic geography. As emphasized in Lucas (1988), urban formation and economic growth should be analyzed within a compact framework (see also, Lucas and Rossi-Hansberg, 2002). This study contributes to the literature of spatial growth with microeconomic foundation by following the main economic theory with endogenous capital.

As far as spatial configuration is concerned, this study is based on neoclassical urban economics. It is generally agreed that the work of Alonso (1964) has played the role of the key model in the development of neoclassical urban economics. There is a large number of research papers which extend and generalize the Alonso model (Imai, 1982; Fujita and Thisse, 2002; Berliant et al., 2002; Forslid and Ottaviano, 2003; Robert-Nicound, 2005; Nocco, *et. al.*, 2017; and Fosgerau, 2018). In his seminal bid-rent theory, Alonso adapted von Thünen's analytical framework on an urban context. He replaced the central market with a central business district (CBD). Papageorgiou and Pines (1999) point out that Alonso's modern spatial analysis was based on microeconomic theory. The approach pioneered modern urban economics. The Alonso model is concerned with urban land use and market land rent. We will follow the Alonso model by accepting the assumption of monocentric city in which all economic activities are concentrated in the CBD. The model shows how residential land use is formed around the CBD. The Alonso model endogenously determines the density of land use, the land rent, and the equilibrium locations of the population.

This study constructs a spatial growth model of endogenous capital accumulation under perfect competition and monopolistic competition. We unify the basic models in three different economic theories. They are respectively neoclassical growth theory, neoclassical urban economics, and new growth theory. The integration is conducted by using Zhang's the concept of disposable income and utility function. By the way, this study extends Zhang's growth model with monopolistic competition (Zhang, 2018) and Zhang's urban development model of endogenous capital (Zhang, 1994, 2007). The rest of the paper is carried out as follows. Section 2 develops the urban dynamic model with residential distribution under perfect and imperfect competition. Section 3 analyzes properties of dynamic urban economy and simulates the dynamics of economic and urban structural changes. Section 4 carries comparative dynamic analysis with regards to exogenous changes in some parameters. Section 5 is the conclusion of the study.

The Model

The economy has not trade with outside world. It is isolated and there is no migration. There are two kinds of economic activities in the supply side: to produce a final good and to produce varieties of differentiated middle products. Middle products are used as intermediate inputs. The final goods sector supplies a homogenous capital goods, which can be invested as capital goods and consumed as final consumer goods. It is like the commodity produced in the Solow model, which can be invested as capital good and consumed as consumer good. This sector perfectly competitive. The intermediate inputs sector is characterized by imperfect competition. Each product is differentiated from the other products. They are not perfect substitutes. Each firm maximizes its profit with the prices charged by other firms as given. It is assumed that each firm has some degree of market power. We measure market power by the terms and conditions of demand and supply equilibrium it has. The residential distribution follows the traditional residential land-use model with linear urban area. This feature is accepted by Beckman (1969; see also Beckmann and Papageorgiou, 1989; Anas, 1990; and Tabuchi and Thisse, 2002). The national economy is developed over on a linear space. The space is composed of the residential area and the CBD. The population is homogenous. People work only in the CBD. All the workers make trips between the CBD and their dwelling sites. We neglect any internal complexity of the CBD, considering it as identical space without any transport cost involved within the CBD. The household resides at only one location. All the households have the equal level of utility in the isolated city. The residential area configurated as a finite strip of land with unit width. The CBD is located at one end of the city. The CBD has the two sectors where workers work. The assumed urban configuration will not change the main conclusion, for instance, if we locate the CBD at the center of the linear system. We use final goods to serve as numeraire. Let us define:

$\bar{N}$ − the fixed population of the city;
L − the land of the urban economy;
$F(t)$ – output level of the final goods sector at t;
$K(t)$ and $\tilde{N}(t)$ − the capital stocks and labor force of the final goods sector at time t;
$r(t)$ – the rate of interest; and
$w(t)$ – the wage rates.

The Production of Final Product

The final goods sector is capital goods sector as in the Solow model. Let $F(t)$, $K(t)$, $\tilde{N}(t)$, and $X(t)$ represent the output level of the final goods sector, level of capital input, level of labor input, and level of aggregate input of intermediates. The (aggregate) input of intermediate inputs of the sector $X(t)$ is given as follows:

$$X(t) = \int_0^n x^\theta(t, \omega) \, d\omega, \; 0 < \theta < 1, \; (1)$$

where n is the number of varieties of middle products available at time t, $x(t, \omega)$ is the input of middle product ω, and θ is a parameter? The production function of the final goods sector is specified as follows:

$$F(t) = A K^\alpha(t) \tilde{N}^\beta(t) X^\gamma(t),$$
$$0 < \alpha, \; \beta, \; \alpha + \beta, \; \gamma = \frac{1 - \alpha - \beta}{\theta} < 1, \; (2)$$

in which α, β, A, and γ are coefficients. The economy has constant returns to scale with fixed n, but increasing returns with varying n. The function implies that technical efficiency rises in the degree of specialization. This study takes n as given. This implies that, the firms the intermediate inputs sector get positive profits. In the literature of new economic geography, n is often treated as an endogenous variable, by the monopolistic competition condition that firms get zero profit.

Physical capital has a fixed depreciation rate, denoted by δ_k. We denote $p(t, \omega)$ the price of middle good ω. We have the profit of the final goods sector as:

$$\pi_0(t) = F(t) - (r(t) + \delta_k) K(t) - w(t) \tilde{N}(t) - \int_0^n p(t, \omega) x(t, \omega) \, d\omega.$$

The marginal conditions are:

$$r_\delta(t) = \frac{\alpha F(t)}{K(t)}, \quad w(t) = \frac{\beta F(t)}{\tilde{N}(t)}, \quad p(\omega, t) = \frac{\gamma \theta x^{\theta-1}(\omega, t) F(t)}{X(t)}, \quad (3)$$

in which $r_\delta(t) \equiv r(t) + \delta_k$. The share of factor $X(t)$ is $\gamma F(t)$. We introduce:

$$z(t) \equiv \frac{r_\delta(t)}{w(t)} = \frac{\tilde{N}(t)}{\bar{\beta} K(t)},$$

where $\bar{\beta} \equiv \beta/\alpha$. From (2) and (3) we get:

$$K(t) = \Lambda(t) X^{\frac{1}{\theta}}(t), \quad \tilde{N}(t) = \bar{\beta} z(t) K(t), \quad (4)$$

where

$$\Lambda(z(t), Z(t)) \equiv \left(\frac{\alpha A \bar{\beta}^\beta z^\beta(t)}{r_\delta(t)} \right)^{1/(\theta \gamma)}.$$

Conditions (3) also imply:

$$p(\omega, t) = \frac{\gamma \theta r_\delta(t) x^{\theta-1}(\omega, t) K(t)}{\alpha X(t)}. \quad (5)$$

Substituting (4) into (5) yields:

$$x(\omega, t) = \tilde{\Lambda}(t) p^{-\varepsilon}(\omega, t), \quad (6)$$

where

$$\tilde{\Lambda}(t) \equiv \left(\frac{\gamma \theta r_\delta(t) X^{(1-\theta)/\theta}(t) \Lambda(t)}{\alpha} \right)^\varepsilon, \quad \varepsilon \equiv \frac{1}{1 - \theta}.$$

We have the share of variety ω in the total value of middle goods:

$$\varphi(t, \omega) \equiv \frac{x(t, \omega) p(t, \omega)}{\int_0^n x(t, \mu) p(t, \mu) d\mu}. \quad (7)$$

Insert (6) in (7)

$$\varphi(\omega, t) = \frac{p^{1-\varepsilon}(\omega, t)}{\int_0^n p^{1-\varepsilon}(\mu, t) \, d\mu}. \quad (8)$$

The Middle Goods Sector

We apply Dixit and Stiglitz (1977) to model the middle goods sector. At t middle goods are produced under monopolistic price competition. The profit is the product of profits per unit of product by the share of the market. The producer of variety ω makes decision on $p(\omega)$ to optimize the profit as follows:

$$\pi(\omega, t) = [p(\omega, t) - a_N w(t)] \frac{\varphi(\omega, t) \gamma F(t)}{p(t, \omega)}.$$

where a_N is the unit labor requirement in producing intermediates? Insert (8) in the above equation:

$$\pi(\omega) = [p(\omega) - a_N w(t)] \frac{p^{-\varepsilon}(\omega, t) \gamma F(t)}{\int_0^n p^{1-\varepsilon}(\mu, t) \, d\mu}. \quad (9)$$

From the first-order condition (i.e., $\partial \pi / \partial p = 0$), we get the fixed-markup pricing rule:

$$\theta p(\omega, t) = a_N w(t). \quad (10)$$

From (10) we observe that the price is equal across variety. The symmetry implies that all the firms charges the same price. From (10) and (9), the profit per firm is given by:

$$\pi(t) = \frac{(1 - \theta)\, \gamma\, F(t)}{n}. \quad (11)$$

The sector's total profit is $n\,\pi(t)$. From (6), we also have $x(\omega, t)$ independent of ω. Hence from (1), we have:

$$X(t) = n\, x^{\theta}(t). \quad (12)$$

Consumer Behavior

We use Zhang's approach to modeling behavior of households (e.g., Zhang, 2005, 2008). We introduce location-dependent variables:

$\widetilde{\omega}$ − the distance from the CBD to a dwelling site in the urban area, $0 \le \widetilde{\omega} \le L$;

$\bar{R}(\widetilde{\omega}, t)$ − the land rent at location $\widetilde{\omega}$;

$\bar{k}(\widetilde{\omega}, t)$, $c(\widetilde{\omega}, t)$ and $s(\widetilde{\omega}, t)$ − the wealth, consumption level of final goods, saving of the household at $\widetilde{\omega}$;

$T_h(\widetilde{\omega})$ and $\Gamma(\widetilde{\omega})$ – the leisure time and travel time;

$\tilde{n}(\omega, t)$ and $\tilde{L}(\widetilde{\omega}, t)$ − the residential density and the lot size of the household at $\widetilde{\omega}$.

We have the following relation between $\tilde{n}(\omega, t)$ and $\tilde{L}(\widetilde{\omega}, t)$:

$$\tilde{n}(\widetilde{\omega}, t) = \frac{1}{\tilde{L}(\widetilde{\omega}, t)}, \quad 0 \le \widetilde{\omega} \le L. \quad (13)$$

In this study we assume that the work time is fixed for each household. The household chooses between leisure and travel time. Let T_0 the time available for travel and leisure. We have:

$$T_h(\widetilde{\omega}) = T_0 - \Gamma(\widetilde{\omega}).$$

It should be noted that some studies consider transportation cost dependent on income (e.g., Train and McFadden, 1978, Rietveld et al, 2003, and De Palma et al, 2005). For simplicity of analysis, we only consider time on travels as the key factor in determining residential location.

The total land revenue $R(t)$ of the land rents is:

$$R(t) = \int_0^L \bar{R}(\widetilde{\omega}, t)\, d\widetilde{\omega}\,. \quad (14)$$

Land revenue can be shared indifferent ways. As in Fujita (1999), we assume the land revenue is equallyshared among the households (see also Fujita and Thisse, 2002). The household obtain the land income $\bar{r}(t)$ as:

$$\bar{r}(t) = \frac{R(t)}{\bar{N}}. \quad (15)$$

The household at $\widetilde{\omega}$ has the current income at time t as follows:

$$y(\widetilde{\omega}, t) = r(t)\, k(\widetilde{\omega}, t) + w(t) + \bar{r}(t) + \bar{\pi}(t), \quad 0 \le \widetilde{\omega} \le L, \quad (16)$$

where $\bar{\pi}(t) = n\,\pi(t)/\bar{N}$ is the profit that the household receives? It is assumed that transactions of wealth is conducted freely and instantaneously. The income available for consuming, saving, and travelling is equal to the value of wealth and the current income. The disposable income equals:

$$\hat{y}(\widetilde{\omega}, t) = y(\widetilde{\omega}, t) + \bar{k}(\widetilde{\omega}, t). \quad (17)$$

The representative household's utility welfare is related to the lot size, consumption level of final goods, leisure time, and savings.

To simplify the modelling, let us omit possible pecuniary travel costs. The household at $\widetilde{\omega}$ distributes the disposable income between the lot size $\tilde{L}(\widetilde{\omega}, t)$, saving $s(\widetilde{\omega}, t)$, and consumption $c(\widetilde{\omega}, t)$. We have the budget constraint as follows:

$$c(\widetilde{\omega}, t) + \bar{R}(\widetilde{\omega}, t)\, \tilde{L}(\widetilde{\omega}, t) + s(\widetilde{\omega}, t) = \hat{y}(\widetilde{\omega}, t). \quad (18)$$

The equation implies that the consumers' disposable personal income is used up by consuming and saving. Choice of residential location is affected by different environmental conditions. One might prefer

to live less dense area. We consider location choice dependent on physical environment such as open space and noise pollution. It is also related to social environmental quality. There are a variety of externalities in the literature of urban economics (Henderson, 1974; Upton, 1981; and Helmers, 2019). Zhang (1993) proposed a spatial equilibrium model with amenity. The importance of amenity is also well recognized in works (e.g., Glaeser et al, 2001; Liu, et al., 2018; and Tivadar and Jaye, 2019). This paper takes account of amenity in analyzing the consumer location decision. We consider amenity related to residential density. The amenity function $\theta(\widetilde{\omega}, t)$ at $\widetilde{\omega}$ is taken on the following form:

$$\tilde{\theta}(\widetilde{\omega}, t) = \tilde{\theta}_0 \, \tilde{n}^a(\widetilde{\omega}, t), \ \tilde{\theta}_0 > 0. \ (19)$$

The function $\tilde{\theta}(\widetilde{\omega}, t)$ means that the amenity at $\widetilde{\omega}$ is dependent on the residential density. When $a = 0$, amenity is constant and independent of space.

It is assumed that the representative household's utility level at $\widetilde{\omega}$ $U(\widetilde{\omega}, t)$ is as follows:

$$U(\widetilde{\omega}, t) = \tilde{\theta}(\widetilde{\omega}, t) \, T_h^\sigma(\widetilde{\omega}) \, c^{\xi_0}(\widetilde{\omega}, t) \, \tilde{L}^{\eta_0}(\widetilde{\omega}, t) \, s^{\lambda_0}(\widetilde{\omega}, t), \ \sigma, \xi_0, \eta_0, \lambda_0 > 0, \ (20)$$

in which σ, ξ_0, η_0, and λ_0 are respectively propensities to enjoy leisure time, to consume final goods, to use lot size, and to own wealth, respectively. Maximizing $U(\widetilde{\omega}, t)$ subject to the budget constraint (17) yields:

$$c(\widetilde{\omega}, t) = \xi \, \hat{y}(\widetilde{\omega}, t), \ \tilde{L}(\widetilde{\omega}, t) = \frac{\eta \, \hat{y}(\widetilde{\omega}, t)}{\tilde{R}(\widetilde{\omega}, t)}, \ s(\widetilde{\omega}, t) = \lambda \, \hat{y}(\widetilde{\omega}, t), \ (21)$$

in which

$$\xi \equiv \rho \, \xi_0, \ \eta \equiv \rho \, \eta_0, \ \lambda \equiv \rho \, \lambda_0, \ \rho \equiv \frac{1}{\xi_0 + \eta_0 + \lambda_0}.$$

Saving is equal to dissaving. That is:

$$\dot{\bar{k}}(\widetilde{\omega}, t) = s(\widetilde{\omega}, t) - \bar{k}(\widetilde{\omega}, t), \ 0 \le \widetilde{\omega} \le L. \ (22)$$

Equilibrium of Free Population Mobility

To simplify the analysis, we consider implies zero cost and no delay in changing location. we are focused on the case that all households have equal level of utility within the city at any point of time. The condition is represented by:

$$U(\widetilde{\omega}_1, t) = U(\widetilde{\omega}_2, t), \ 0 \le \widetilde{\omega}_1, \widetilde{\omega}_2 \le L. \ (23)$$

The isolated state's population is distributed over the city:

$$\int_0^L \tilde{n}(\widetilde{\omega}, t) \, d\widetilde{\omega} = \bar{N}. \ (24)$$

Let $N_x(t)$ stand for the labor force used by the final goods sector. According to the definitions, we have:

$$N_x(t) = n \, a_N(t) \, x(t).$$

The full employment of labor force implies:

$$\tilde{N}(t) + N_x(t) = \bar{N}. \ (25)$$

The total consumption of final goods $C(t)$ is the sum of the households' consumption:

$$\int_0^L c(\widetilde{\omega}, t) \, \tilde{n}(\widetilde{\omega}, t) \, d\widetilde{\omega} = C(t). \ (26)$$

The output level of the final goods sector equals the total consumption of final goods and net saving:

$$C(t) + S(t) - K(t) + \delta_k \, K(t) = F(t), \ (27)$$

where $S(t)$ is the total saving?

$$\int_0^L s(\widetilde{\omega}, t) \, \tilde{n}(\widetilde{\omega}, t) \, d\widetilde{\omega} = S(t).$$

We have capital fully employed as:

$$\int_0^L \bar{k}(\tilde{\omega}, t)\, \tilde{n}(\tilde{\omega}, t)\, d\tilde{\omega} \; = \; K(t). \quad (28)$$

We thus constructed the urban development model with endogenous capital accumulation and residential location. Firms are assumed to maximize their profits, households are assumed to maximize their utilities, markets are perfectly and imperfectly competitive, and all the input factors are assumed to be fully employed. It is straightforward to see that model is built by synthesizing the Alonso model, the Solow model, and the Dixit-Stiglitz model.

Dynamics of Spatial Equilibrium

The previous section proposed a development model with an integration of the Solow one-sector growth model, the Alonso residential distribution model, and the Dixit-Stiglitz model with a single analytical framework. The economic development mechanism is due to neoclassical capital accumulation in markets of perfect and monopolistic competition. We give a lemma to provide a computational program for describing the dynamics of the economic system.

Lemma

The following differential equation determines the motion of the economic system:

$$\dot{\bar{k}}(t) \; = \; \Phi\!\left(\bar{k}(t)\right), \quad (29)$$

where Φ is a function of $\bar{k}(t)$ defined in the Appendix? Moreover, all the variables are represented as functions of $\bar{k}(t)$ by a computational procedure:

$z(t)$ by (A19) $\to x(t)$ by (A18) $\to K(t) = \bar{k}(t)\,\bar{N} \to \tilde{N}(t)$ by (A17) $\to X(t)$ by (12) $\to K(t)$ by (25) $\to F(t)$ by (2) $\to r(t)$ and $w(t)$ by (3) $\to p(t)$ by (10) $\to \hat{y}(t)$ by (A15) $\to p(t)$ by (11) $\to \pi(t)$ by (11) $\to \bar{\pi}(t) = \mathrm{n}\,\pi(t)/\bar{N} \to c(t)$ by (21) $\to s(t)$ by (21) $\to \tilde{n}(0)$ by (A12) $\to n(\tilde{\omega})$ by (A11) $\to \tilde{L}(\tilde{\omega}, t)$ by (13) $\to \tilde{r}(t)$ by (A14) $\to \tilde{R}(\tilde{\omega})$ by (A9) $\to N_x = n\, a_N\, x \to \tilde{N} = \bar{N} - N_x \to R(t)$ by (A13) $\to U(t)$ by (20).

We show the Lemma in the Appendix. As the expressions are tedious, we simulate dynamic behavior of the economy. The parameter values are specified as follows:

$$\bar{N} = 100, \; L = 5, \; n = 200, \; \theta = 0.6, \; \alpha = 0.3, \; \beta = 0.4, \; a = 0.1,$$
$$\xi_0 = 0.15, \; \eta_0 = 0.05, \; \lambda_0 = 0.6, \; \sigma = 0.25, \; \tilde{\theta}_0 = 3.1, \; a_N = 0.2,$$
$$v = -0.05, \; \delta_k = 0.05. \quad (30)$$

The population is 100. The length of the city is 5. It should be noted that specified values of the parameters will not affect our analysis essentially as we will examine how shifts in parameter values affect the movement of the national economy. The household allocates 75 percent of the disposable income for saving and the rest for consuming. As demonstrated in the Appendix, the labor distribution and the output of each firm in the intermediate inputs sector are invariant in time. Under (30) these variables values are given as follows:

$$\tilde{N} = 38.8, \; N_x = 61.2, \; x = 0.44, \; X = 121.6.$$

The final goods sector employs most of the labor force. The residential density and the amenity are location-related but are invariant in time as illustrated in Figure 1.
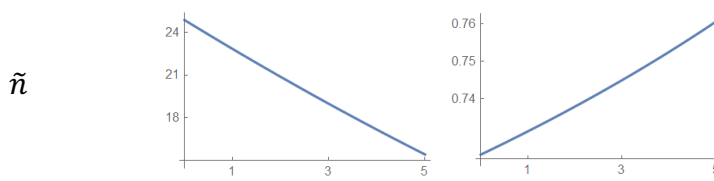
$\tilde{n}$

Figure 1: Residential and Amenity Distribution

The residential density falls in distance, which also means that the household has a larger house further away from the CBD. The amenity rises over space. People live far away from the CBD has better environment. As shown late, the rent falls as the dwelling site is further away from the CBD. We specify the initial condition as follows:

$$\bar{k}(t) = 42.$$

The simulation result for time-dependent variables is plotted in Figure 2. From the initial state, the output of the final goods sector falls. The national capital ($= \bar{k} \bar{N}$) falls. The interest rate rises and the income from land rent falls. The price of intermediate inputs falls. The consumption level also falls over time.
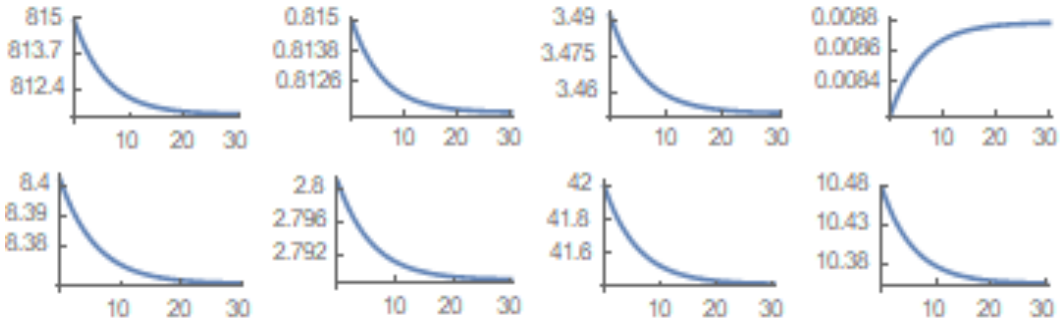

Figure 2: The Motion of the Time-Dependent Variables

The land rent is dependent on time and space as illustrated in Figure 3. The land rent falls in distance and in time.
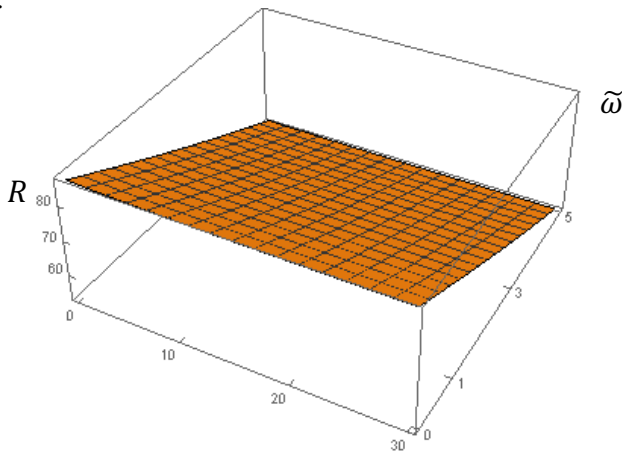

Figure 3: The Motion of the Rent Over Time and Space

We easily identify the following equilibrium variable values:
$$F = 811.5, \quad \pi = 0.81, \quad \bar{r} = 3.45, \quad r = 0.009, \quad w = 8.37, \quad p = 2.79,$$
$$\hat{y} = 55.2, \quad \bar{k} = 41.4, \quad c = 10.4.$$

We calculate that the eigenvalue is -0.168. We conclude that there is a stable unique equilibrium point. This guarantees the validity of comparative dynamic analysis in the next section.

Comparative Dynamic Analysis

This section deals with effects of shifts in some parameters on the urban economy. As the urban economy contains a stable unique equilibrium point, it is valid to conduct comparative dynamic analysis. We define a symbol $\bar{\Delta} x$ to represent the change rate of a variable x in percentage caused by shifts in the value of a parameter.

A Rise in Variety of Intermediate Inputs

We first examine what will occur in the spatial economy when the variety of intermediate inputs rises as follows: $n: 200 \Rightarrow 205$. We consider a rise in the variety by innovation and introduction of new products. The effects on the constant variables are listed as:

$$\bar{\Delta} x = -0.024, \qquad \bar{\Delta} X = 0.01, \qquad \bar{\Delta} \tilde{N} = \bar{\Delta} N_x = 0.$$

Each firm of the intermediate inputs sector reduces its output but the aggregate input of the final sector is enhanced. The labor distribution is invariant. The residential distribution and amenity level are invariant, that is, $\bar{\Delta} \tilde{n}(\tilde{\omega}) = \bar{\Delta} \tilde{\theta}(\tilde{\omega}) = 0$. The simulation result for the time-dependent variables is plotted in Figure 2. The output of final goods sector and total wealth rise. The profit per firm falls. The household's land rent income rises. The wage rate and the price of intermediate inputs are increased. The interest rate is increased initially but is changed slightly in the long term. The consumption and wealth levels rise.
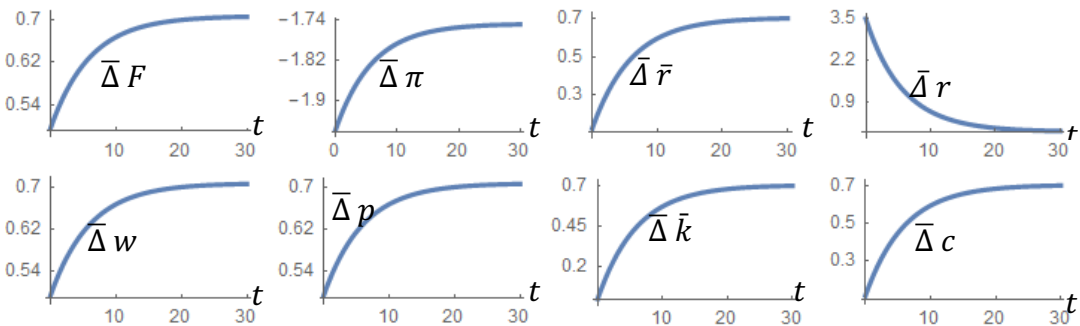

Figure 4: A Rise in the Variety of Intermediate Inputs

From $\tilde{R}(\tilde{\omega}, t) = \eta \, \tilde{n}(\tilde{\omega}) \, c(t)/\xi$ where $\eta \, \tilde{n}(\tilde{\omega})/\xi$ are not affected, we see that the rent at any position changes at the same rate in time as $c(t)$.

A Rise in the Elasticity of Substitution Between two Varieties

We now deal with the impact of the following rise in the elasticity of substitution between two varieties on the economy: $\theta = 0.6$ to 0.605. The effects on the time-dependent variables are:

$$\bar{\Delta} x = 0.006, \qquad \bar{\Delta} X = -0.0001, \qquad \bar{\Delta} \tilde{N} = -0.003, \qquad \bar{\Delta} N_x = 0.002.$$

The output of per firm is enhanced slightly, but the aggregate input is slightly reduced. The labor force is moved from the final goods sector to the intermediate inputs sector. The simulation result for the time-depenent variables is plotted in Figure 5. Each firm of the middle goods sector has less profit. The output of the final goods sector sector is reduced. The land rent income is reduced. The interest rate falls. The wage rate falls. The household has less wealth and consumes less.
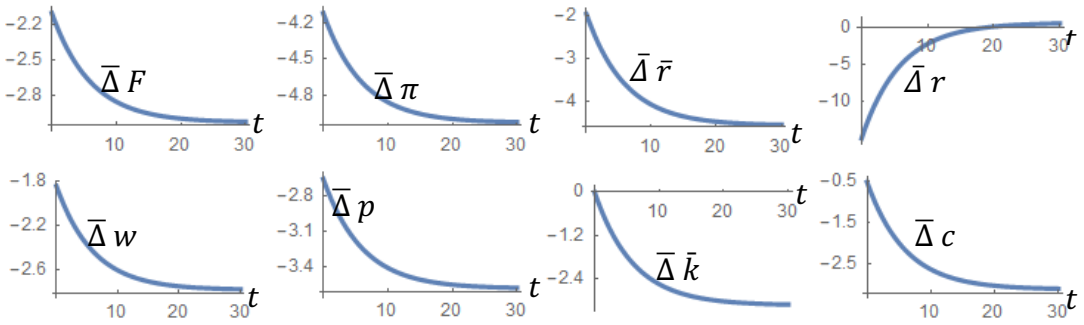
Figure 5: A Rise in the Elasticity of Substitution Between Two Varieties

It is shown that the residential density and amenity are not affected. The land rent changes at each location in the same rate as the consumption level.

Increases in the Population or Propensity to Consume Leisure

We first change the population as follows: $\bar{N} = 100 \Rightarrow 102$. We have the following impact on the variables:

$$\bar{\Delta} F = \bar{\Delta} \pi = 2, \ \bar{\Delta} w = \bar{\Delta} r = \bar{\Delta} p = \bar{\Delta} c = \bar{\Delta} \bar{r} = \bar{\Delta} \bar{k} = 0, \ \bar{\Delta} x = 0.02,$$
$$\bar{\Delta} X = 0.012, \ \bar{\Delta} \tilde{N} = \bar{\Delta} N_x = 0.02, \ \bar{\Delta} \tilde{n}(\tilde{\omega}) = 2, \ \bar{\Delta} \tilde{\theta}(\tilde{\omega}) = -0.2.$$

We see that as the population is increased, there are increases in the output of the final goods sector, the profit and output per firm in the intermediate inputs sector. The residential density rises and the amenity falls.

We now raise the propensity to enjoy leisure as: $\sigma = 0.25$ to 0.26. We see that the residential density and land rent near the CBD are increased, while the residential density and land rent far away from the CBD are reduced. The amenity near the CBD is reduced, while the amenity far away from the CBD is improved.
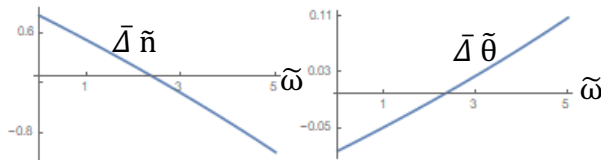

Figure 6: A Rise in the Propensity to Consume Leisure

A Rise in the Labor Cost of the Intermediate Inputs Sector

We now study the effects that the following rise in the labor cost of the intermediate inputs sector: $a_N: 0.2 \Rightarrow 0.21$. We have the following effects on the time-independent variables:

$$\bar{\Delta} x = -0.05, \qquad \bar{\Delta} X = -0.03, \qquad \bar{\Delta} \tilde{N} = \bar{\Delta} N_x = 0.$$

The output of each firm and aggregate input of the intermediate inputs sector fall. The labor distribution is not affected. The simulation result for the time-dependent variables is plotted in Figure 6. We see that except the price of intermediate inputs, the values of all the other variables are reduced. We show that the residential distribution and amenity are not affected. The land rent falls in the rate equal to that of the consumption level.
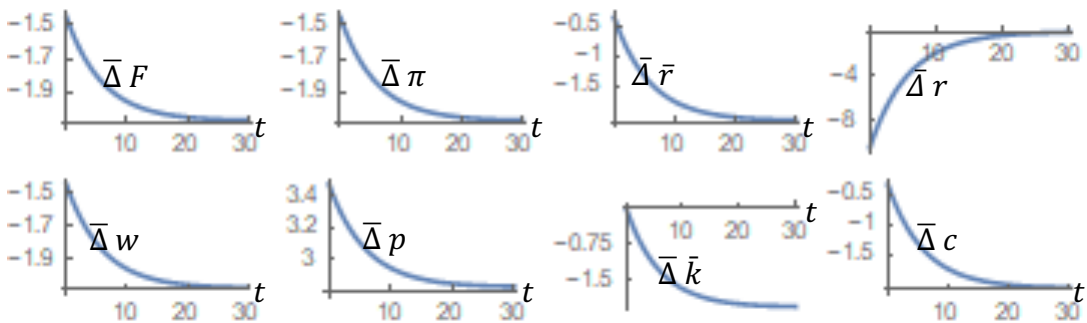
t

Figure 7: A Rise in the Labor Cost of the Intermediate Inputs Sector

A Rise in the Propensity to Save

We study the effects that an increase in the propensity to save: $\lambda 0 = 0.6$ to 6.05. We have the following effects on the time-independent variables:

$$\bar{\Delta} x = -0.05, \qquad \bar{\Delta} X = -0.03, \qquad \bar{\Delta} \tilde{N} = \bar{\Delta} N_x = 0.$$

The output of each firm and aggregate input of the intermediate inputs sector fall. The labor distribution is not affected. The simulation result for the time-dependent variables is plotted in Figure 8. The output of the final goods sector and profit of each firm in the intermediate inputs sector are augmented. The residential distribution and amenity are not affected. The wage rate and price of intermediate inputs are enhanced. The interest rate falls. The wealth level is enhaned. At each point of time the land rent at each location is changed in the same rate as the consumption level.
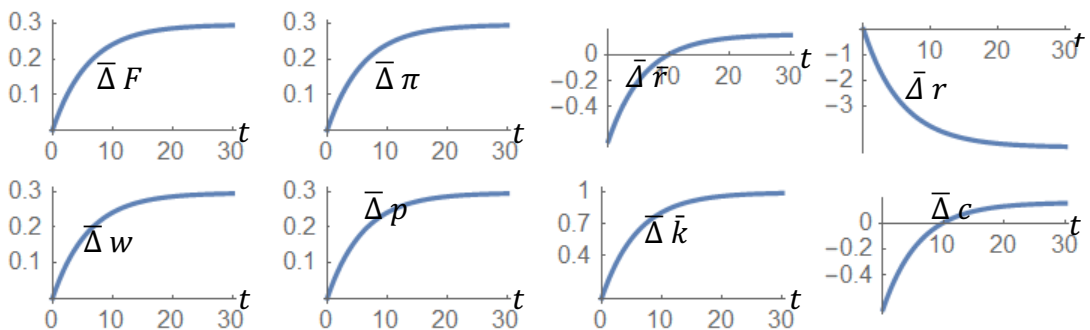

Figure 8: A Rise in the Propensity to Save

Concluding Remarks

This paper was concerned with dynamic relations between economic growth, economic structure, and residential distribution. It developed a spatial dynamic economic model on basis of microeconomic foundation. The paper carried out a unique contribution to the literature of spatial economic dynamics with microeconomic foundation. The model is built by integrating the ideas in three main models in the literature of theoretical economics. They are respectively the Solow one-sector growth model, the Alonso urban residential model, and the Dixit-Stiglitz equilibrium model with imperfect market. We apply the neoclassical economic growth mechanism with perfect competition to model growth, the neoclassical urban residential structural determination to model residential location, and the basic model of modelling imperfect market to have both perfect and imperfect competition in spatial equilibrium structure. The basic economic mechanisms of the three approaches are integrated by using Zhang's approach to household behavior. We first built the economic growth with economic geography. We determine economic equilibrium by simulation. We conducted comparative dynamic analysis to show how changes in different parameters affect residential distribution, economic growth and economic structure. As it

introduces interdependence between multiple factors, the model is built with many strict assumptions. Spatial structure is over simplified as firms may not be located to the CBD. There are different types of households. We see possible generalizations and extensions by referring to the literature of urban economics and growth theory. For instance, following the recent literature on growth with endogenous knowledge, we may model spatial dynamics of economic growth with endogenous wealth, knowledge and population.

Appendix: Proving the Lemma

Inserting $N_x = n\, a_N\, x$ and (22) in (25), we have

$$x = \frac{\bar{N} - \bar{\beta}\, z\, K}{n\, a_N}. \quad (A1)$$

From the definitions we have

$$\hat{y}(\tilde{\omega}) = (1 + r)\, \bar{k}(\tilde{\omega}) + w + \tilde{r} + \bar{\pi}. \quad (A2)$$

Insert $\bar{\pi} = n\, \pi/\bar{N}$ and (11) in (A2)

$$\hat{y}(\tilde{\omega}) = (1 + r)\, \bar{k}(\tilde{\omega}) + W, \quad (A3)$$

in which we use (3) and $\tilde{N} = \bar{\beta}\, z\, K$ and W is a variable independent of location:

$$W(t) = w + \tilde{r} + \frac{(1 - \theta)\, \gamma\, r_\delta\, K}{\alpha\, \bar{N}}.$$

We thus have

$$s(\tilde{\omega}, t) = (1 + r)\, \lambda\, \bar{k}(\tilde{\omega}) + \lambda\, W. \quad (A4)$$

Insert (A4) in (22)

$$\dot{\bar{k}}(\tilde{\omega}, t) + \tilde{r}(t)\, \bar{k}(\tilde{\omega}, t) = \lambda\, W(t), \quad 0 \le \tilde{\omega} \le L, \quad (A5)$$

Where

$$\tilde{r}(t) = 1 - \lambda - \lambda\, r(t).$$

This is a linear equation with time-dependent variables. The solution of (A5) is given by:

$$\bar{k}(\tilde{\omega}, t) = e^{-\int_0^t \tilde{r}(\omega)\, d\omega} \left[\int_0^t e^{\int_0^\omega \tilde{r}(\varepsilon)\, d\varepsilon}\, \lambda\, W(\omega)\, d\omega + \tilde{C} \right], \quad (A6)$$

where $\tilde{C}$ is determined a constant determined by the initial condition. If the wealth is assumed to be initially independent of the residential local, $\bar{k}(\tilde{\omega}, 0) = \bar{k}(0)$, , we have that $\bar{k}(\tilde{\omega}, t)$ is independent of the residential location. That is, all the households in the system have the same level of wealth. This is important for us to solve the model as become evident late on.

From (A3) we see $\hat{y}$ independent of $\tilde{\omega}$. From (21) we conclude that c and s are independent of $\tilde{\omega}$. From (20) and (21) we have:

$$U(\tilde{\omega}) = \tilde{\theta}_0\, T_h^\sigma(\tilde{\omega})\, \xi^{\xi_0}\, \eta^{\eta_0 - a}\, \lambda^{\lambda_0}\, \tilde{R}^{a - \eta_0}(\tilde{\omega})\, \hat{y}^{\eta_0 - a + \lambda_0 + \xi_0}, \quad (A7)$$

where we use (19) and (13). Insert (A7) in $U(0) = U(\tilde{\omega})$ for any

$$\tilde{R}(\tilde{\omega}) = T_R(\tilde{\omega})\, \tilde{R}(0), \quad (A8)$$

Where

$$T_R(\tilde{\omega}) \equiv \left(\frac{T_h(0)}{T_h(\tilde{\omega})} \right)^{\sigma/(a - \eta_0)}.$$

From (21) and (13), we get

$$\tilde{R}(\tilde{\omega}) = \eta\, \hat{y}\, \tilde{n}(\tilde{\omega}). \quad (A9)$$

Equations (A8) and (A9) imply

$$\tilde{R}(0) = \frac{\eta\, \hat{y}}{T_R(\tilde{\omega})}\, \tilde{n}(\tilde{\omega}) = \eta\, \hat{y}\, \tilde{n}(0). \quad (A10)$$

From (A10) we have

$$\tilde{n}(\tilde{\omega}) = T_R(\tilde{\omega}) \, \tilde{n}(0) . \quad (A11$$

Insert (A11) in (24)

$$\tilde{n}(0) = \bar{N} \left(\int_0^L T_R(\tilde{\omega}) \, d\tilde{\omega} \right)^{-1} . \quad (A12)$$

We thus determine the residential distribution by (A12) and (A11). We determine $\tilde{R}(\tilde{\omega}, t)$ by (A9) and (A8). With (A9) and (A11), we have

$$R = \eta \, \hat{y} \, \bar{N} . \quad (A13)$$

By (A13) and (15), we have:

$$\bar{r} = \eta \, \hat{y} . \quad (A14)$$

As $\bar{k}$ is independent of location, we have $K = \bar{k} \, \bar{N}$. Insert (A14) in (A3)

$$\hat{y} = \left(1 + r + \frac{(1 - \theta) \, \gamma \, r_\delta}{\alpha} \right) \frac{\bar{k}}{1 - \eta} + \frac{w}{1 - \eta} . \quad (A15)$$

From (3), we have:

$$w = \frac{\beta \, p \, n \, x}{\gamma \, \theta \, \tilde{N}} , \quad (A16)$$

where we also use (12). Insert $p = a_N \, w/\theta$ in (A16)

$$\tilde{N} = \tilde{a} \, x , \quad (A17)$$

Where

$$\tilde{a} = \frac{\beta \, a_N \, n}{\gamma \, \theta^2} .$$

From $\tilde{N} = \bar{\beta} \, z \, K$ and (A17), we solve

$$x = \frac{\bar{\beta} \, z \, K}{\tilde{a}} . \quad (A18)$$

Equal (A18) and (A1)

$$z = \frac{\alpha}{(\gamma \, \theta^2 + \beta) \, \bar{k}} , \quad (A19)$$

where we use $K = \bar{k} \, \bar{N}$. From (A19) we see that $z \, \bar{k}$ is constant. From $K = \bar{k} \, \bar{N}$ and (A18) we see that x is constant. Equation (A17) implies that the labor distribution is invariant in time. We also have a constant X.

It can be seen that all the variables can be expressed as functions of $\bar{k}$ by the following procedure: z by (A19) $\rightarrow x$ by (A18) $\rightarrow K = \bar{k} \, \bar{N} \rightarrow \tilde{N}$ by (A17) $\rightarrow X$ by (12) $\rightarrow K$ by (25) $\rightarrow F$ by (2) $\rightarrow r$ and w by (3) $\rightarrow p$ by (10) $\rightarrow \hat{y}$ by (A15) $\rightarrow p$ by (11) $\rightarrow \pi$ by (11) $\rightarrow \bar{\pi} = n \, \pi / \bar{N} \rightarrow c$ by (21) $\rightarrow s$ by (21) $\rightarrow \tilde{n}(0)$ by (A12) $\rightarrow n(\tilde{\omega})$ by (A11) $\rightarrow \tilde{L}(\tilde{\omega}, t)$ by (13) $\rightarrow \bar{r}$ by (A14) $\rightarrow \tilde{R}(\tilde{\omega})$ by (A9) $\rightarrow N_x = n \, a_N \, x \rightarrow \tilde{N} = z \, \bar{\beta} \, K \rightarrow R$ by (A13) $\rightarrow U$ by (20).

From the procedure and (22), we have

$$\dot{\bar{k}} = \Phi(\bar{k}(t)) \equiv s(\bar{k}) - \bar{k} . \quad (A20)$$

In summary, we proved the Lemma.

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