Exploring Content-Related Difficulties Faced by Student-teachers of B.Ed Programme in Learning 'Derivation' Concept at Sukkur IBA University

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Abstract: *This study explores student-teacher content-related difficulties in learning the "Derivation" concept in B.Ed. Program of Sukkur IBA University. The qualitative study used exploratory research design and purposive sampling for data collection. Data was collected through interviews and thematically analysed. The study found that most students struggle with derivation, including building a basic concept, understanding the logic behind the definition, differentiating function and variable in Leibniz Notation, understanding word problem words, deriving information from word problems, and choosing an appropriate technique to solve word problems. Most students struggle to understand limited concepts. The current study suggested that student-teachers should be taught by changing teaching methodology from procedural to conceptual, teaching the pre-concept of the main concept, connecting mathematical concepts with real life, and using a problem-solving approach to learn derivation.*

Key Words: Mathematics, Content Related Difficulties, B.Ed. Student-teachers, Derivation

Introduction

Mathematics is an important subject at elementary and secondary level in Pakistan. Mathematics enables to deal and solve the complex problem in real life. In order to make a significant progress learners must be equipped with mathematical skills (Kiani, Malik, & Ahmad, 2012). Mathematics helps to build logic and pattern recognitions in brain. The mathematics develops organizational and problem solving skills of a learner. While learning mathematics students look at the whole problem before solving it. Learners look the possible solution, follow certain steps to solve a mathematical problem and come at conclusion. Furthermore learner evaluate the final answer. Mathematics stimulates the brain to think rationally and it also increases the thinking ability of learner (Thomas, Van Garderen, Scheuermann, & Lee, 2015). The mathematical skills are essential for meeting the needs of increasing complex technological society and skilled labour force. Moreover, children should learn mathematics in order to

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make more progress in STEM (science, technology, engineering, and mathematics) education (National Mathematics Advisory Panel, 2008). Therefore, the Pakistani government has implemented a standard based mathematics curriculum. The National Mathematics Curriculum consists of five standards. This includes number and operations, algebra, measurements and geometry, information management and reasoning and logical thinking (Pakistan, 2006). In addition, each standard includes benchmarks that explain on the standards. The benchmarks describe what students will know and be able to do upon completion of a particular developmental stage. The developmental phases of the national mathematics curriculum are stages one (I-II), two (III-V), three (VI-VIII), four (IX-X), and five (XI-XII) (Ministry of Education, 2006). These outcomes describe what students will know and be able to do for each topic at a specific developmental period. In order to teach at all developmental levels, elementary and secondary level mathematics teachers must be skilled and qualified (Gulzar & Mahmood, 2018).

Mathematics is an important subject at elementary and secondary level in Pakistan. Mathematics enables us to deal and solve complex problems in real life. In order to make significant progress learners must be equipped with mathematical skills (Kiani, Malik, & Ahmad, 2012).Keeping in view the importance of mathematics from a policy and curricular perspective, the Department of Education at Sukkur IBA University offers General Mathematics, Mathematics Content and Pedagogy I and Mathematics Content and Pedagogy II courses. The student-teachers face many difficulties in the learning of mathematics. The majority of student-teachers hardly pass out the subject of calculus. More specifically, they feel difficulty in understanding the concept of Derivation. Therefore, this study was aimed at exploring the content related difficulties faced by undergraduate B.Ed. students in learning the concept of derivation.

Problem Statement

The preparation of teachers plays a crucial role in enhancing the quality of instruction and education. The emphasis on the preparation of qualified mathematics teachers has been highlighted in all national education policies of Pakistan, specifically the National Education Policy of 2009. Furthermore, the implementation of the United States Agency for International Development (USAID) and the Higher Education Commission of Pakistan (HEC) 4-year teacher-led preparation program, which places particular emphasis on equipping science graduates with both content and pedagogical knowledge, was a central component of the B.Ed. initial teacher education program (USAID, 2008). During the course of bachelor of Education (B.Ed.), the undergraduate B.Ed students have to choose a particular area for their subject specialization (e.g Mathematics and Physics, Biology and Chemistry).In addition, they can select their specialization in degree either elementary or secondary. Those who select B.Ed. secondary and obtained mathematics as specialization have to study few courses related to mathematics specialization in four years bachelor in education B.Ed. programme in the Department of Education at Sukkur IBA University. The B.Ed. Programme consists of foundational, compulsory and content related course of mathematics i.e General Mathematics, Mathematics Content and Pedagogy I and Mathematics Content and Pedagogy II courses. The student-teachers were facing many difficulties in the learning of mathematics. The majority of student-teachers fail in the subject of calculus. More specifically, they feel difficulty in understanding the concept of Derivation.

Research Objectives

The main objectives of the study are;

1. To explore the content related difficulties of student-teachers in learning the concept of Derivation in the course of calculus?
2. To investigate the content related difficulties of student-teachers while studying the limit

3. To discover the difficulties faced by student teachers in solving word problems in calculus

Research Questions

In order to identify the problem and achieve the objectives of the study, the researchers have formulated following three questions for the study.

1. Which content related difficulties are faced by B.Ed. students while learning the concept of derivation in the course of Calculus?
2. Which content related difficulties occur when B.Ed. students study limit?
3. What are the difficulties faced by student teachers in solving word problems in calculus?

Literature Review

Mathematics has remained the significant subject at levels of education in Pakistan. It helps to coup up with the complex problems that occur in day to day life Mathematics helps to build logic and pattern recognitions in brain. It also develops logical and problem solving skills of the learners. So, the learners must be taught mathematics rigorously (Kiani et al., 2012). The mathematics follows the procedural steps in solving any problem and come up with the right answer. Moreover, Mathematics stimulates the brain to think rationally and logically. It has become essential to learn mathematical skills in order to meet the needs of technological advanced society (Jojo, Maharaj & Brijlall, 2013).Furthermore, the National Mathematics Advisory Panel (2008) has established a correlation between the advancement in STEM fields, namely science, technology, engineering, and mathematics, and the learning of mathematical knowledge. The Government of Pakistan, specifically the Ministry of Education and Training, has implemented a standardized curriculum for mathematics (Ministry of Education, 2006). The National Curriculum of Mathematics consists of five standards. The academic disciplines encompassed in the curriculum are number and operations, algebra,

measurements and geometry, information handling, and reasoning and logical thinking (Government of Pakistan, 2006).Furthermore, the standards were divided into specific benchmarks for different grade levels with focused learning objectives as well as the benchmarks are developed on the basis of developmental stages in National Curriculum of Mathematics (NCM).The five stages includes a stages one (I-II), stage two (III-V) ,stage three (VI-VIII) ,stage four (IX-X) ,and stage five (XI-XII) (Khan, Farooqi, & Mehmood, 2018). Further, each developmental stage consists of students learning outcomes against every standard which evaluates students' progress through topic wise (Ministry of Education, 2006).

According to Aspinwall & Presmeg (1997) students thinking ability is hindered by incorrect mathematical image. Due to incorrect mathematical image students keep ordinary learning with them and move towards subsequent concepts. It severely effects on later concepts (Yoon, 2016).In a study conducted by Jones (2018) reported that he involved students with images and understanding of fundamental theorem of calculus. The findings of experiment revealed that students' weak concepts regarding the rate of changes led them to difficulties in understanding concept of anti-derivatives. The study of Jojo, Maharaj & Brijlall (2013) indicated that students retain the certain rules of calculus while neglect other rules. This statement is proposed in the light of findings which showed that when students were attempting to solve a non-routine calculus graphing problem they were finding it difficult to coordinate the information.

Furthermore, the discontentment with calculus is observed in numerous nations. The report published by the London Mathematical Society in 1992 acknowledges the challenges associated with university mathematics and proposes the reduction and reorganization of its content. In United States it is seen that 600000 takes the college calculus course, only 46% passed by obtaining D grade or above (Tall,1992).Further, Tall states that no matter how the calculus is taught but there are certain

difficulties which cause problems. Some difficulties of limit are highlighted are here. First students feel difficulty in the language ie. "Limit tends to" "approaches to" "as small we please" these have colloquial meaning which mismatch with the formal concepts. Secondly limit process is not solved by common arithmetic and algebra so the concept of Infinite seems a mystery. The third difficulty is students assume that "N" getting arbitrary large gives the idea of infinite numbers. The fourth difficulty is that students often think that whether limit can actually be reached. Like limit students also face certain difficulties in derivation. In addition, student also face difficulties in comprehending the word problems related to derivation and limit (Nursyahidah & Albab, 2017). As for as the teaching of calculus in B.Ed. programmes concerned many students do not pass the calculus course which is the problem for many students to graduate on time. Therefore, this study is focused to explore the content related difficulties of student-teachers in calculus subject taught in B.Ed. programme at Sukkur IBA University.

Learning difficulties in understanding the calculus and more specifically the concept of derivation includes; Leibniz notation dy/dx is one of the important concept in derivation. The lack of understanding of this concept greatly affect the conceptual learning of derivation. Students often confuse whether it is an indivisible symbol or a fraction. Students did not distinguish relationship between dx in dy/dx and dx in $\int f(x)\, dx$, converting real word problems into calculus formulation, absorbing complex idea in limited time, lack of visual representation of concepts and rote memorization (Hamid, Idris & Tapsir, 2020)

Furthermore, students perform poor in calculus because of basic algebraic skills such as factorization, solving equation, handling operation involving number and indices (Luneta & Makonye, 2010). Further, they added that poor algebraic skills has direct effect on calculus. Tarmizi (2010) states that students do not understand the concept of function and limit as a result as a result they misunderstand

the main concepts of calculus i.e. differentiation, continuity and anti-derivatives. The concept image is crucial for underacting mathematical concept. states that difficulties in derivation occurs due to the lack of understanding of basic of calculus. It is suggested that considerable attention should be given to explanations and illustrations regarding the meaning of limit and continuous function.

The existing body of literature on calculus indicates that students tend to acquire a higher level of procedural understanding compared to conceptual understanding when it comes to applying the chain rule. The graph effectively conveys several significant concepts and provides valuable information that aids in the resolution of a mathematical problem. Graphical representations and interpretations play a pivotal role in the analysis and resolution of calculus problems. According to a study conducted by Orhun (2012), students encounter challenges when attempting to establish a connection between the graph of a derived function and the original function. The utilization of mathematical terminology is lacking in their depiction of the graph of a derivative function. Furthermore, it is evident that students are deficient in certain skills that are necessary for achieving proficiency in graphical representation. The skills encompassed in this context pertain to the comprehension of fundamental principles underlying graphs, the ability to engage in critical thinking with respect to graphs, and the capacity to construct persuasive arguments based on graph analysis. The utilization of graphical interpretation has experienced significant growth in the field of mathematics education. Mathematicians are placing increased emphasis on the significance of derivation across various disciplines and have proposed a methodology that incorporates graphical operations.

Horvath (2008) asserts that the concept of the chain rule poses a significant challenge when attempting to communicate it to calculus students. The expression of the chain rule in symbolic form can be challenging, even after

one has developed an understanding of it. The majority of students lack comprehension regarding the origin of the outcome. The articulation of the chain rule poses a considerable challenge, thereby impeding students' ability to accurately apply it. According to Webster (1978), the chain rule holds significant importance as a fundamental and indispensable concept within the field of calculus. Furthermore, the topic of the chain rule and its practical applications comprises a significant portion of the differentiation chapter within a Thomas calculus textbook. Further investigation is warranted to delve into the significance and intricacy of the chain rule, as it has been observed that students encounter difficulties in comprehending this mathematical concept and recognizing its relevance within the curriculum. Despite its significance in the curriculum, the chain rule has received limited attention in the field of mathematics educational research.

Visualization is a widely recognized and effective method of problem representation in the context of mathematical problem-solving (TOKGÖZ, 2012). According to Garderen and Montague (2003), it has been argued that an effective problem solver initially constructs a problem representation in order to enhance comprehension. A research investigation was undertaken to analyse the problem-solving strategies and identify the learning challenges encountered by students in the process of acquiring calculus knowledge. According to the findings of Tarmizi's study (2010), mathematics students commonly perceive calculus as a challenging subject and often possess misconceptions regarding the concept of a function. Similar to the concept of a mathematical function, where a variable y is determined by a variable x through a defined procedure denoted as f(x), the process involves taking x as an input and executing a series of operations to yield the output y. The majority of the students adhered to the prescribed guidelines by performing operations such as expanding brackets and consolidating like terms. Furthermore, it is worth noting that students commonly employ the substitution rule as a preliminary step prior to undertaking the necessary expansion, a skill that is typically acquired through prior instruction. Based on the findings, it is recommended that students should exercise vigilance in their approach to solving mathematical problems. It is recommended that significant emphasis be placed on cultivating students' capacity to formulate problem-solving strategies. Failure to consider these suggestions may result in increased confusion among students, potentially compromising their ability to succeed in mathematics in the future. The literature also highlights the importance of incorporating problem-solving strategies in mathematics education, as these strategies empower learners to effectively solve mathematical problems by following logical steps.

Research Methodology

The study was conducted to explore the content related difficulties faced by undergraduate B.Ed. students in learning derivation. This study employed qualitative exploratory research approach to explore the content related difficulties faced by undergraduate B.Ed. students in learning derivation at Sukkur IBA University. Semi-structured interviews was conducted from the six participants (3 male,3 female) (Blandford, 2013).The participants were selected considering the gender, educational background (Pre- Engineering and Pre- Medical) and previous performance in derivation from the total population of students who were studying derivation. The researchers employed the purposive sampling technique in order to gather data for the study. Purposive sampling refers to the intentional selection of participants based on specific qualities possessed by the individuals (Tongco, 2007).

The validity of the interview guide was done through taking inputs from the three experts. Moreover, the reliability of the interview guide was ensured through triangulation (Galletta, 2013). . The context of the study was initial teacher preparation four years bachelor in education B.Ed. programme in department of Education at Sukkur IBA University, a public funded University in 2020.

For data analysis, the process of data analysis was initiated by transcribing interviews. The interviews were conducted in regional languages (Sindhi and Urdu) and transcribed into English. The obtained data from interviews were organized and categorized into main themes. The findings of this study is emerged from the data obtained from interviewees. The six participants (3 males, 3 females) responded the open ended questions of semi-structured interview. Data obtained from interviews was condensed into three major themes these include key difficulties in limit concept, students' difficulties in derivation and students' difficulties in solving word problems of derivation. These themes addresses research question of this study.

Results and Findings

Misconception in Derivation

Student 3 stated "Well my area of interest is English. When I hear derivation I understand it's meaning from language point of view. I have idea that in derivation we are given a problem and we will derive a new problem". Also student 2 said "Derivation is the process of deriving from something or making a rule from something".

Four out of six students had difficulty in understanding concept of derivation. On student had understood that derivation is the process of deriving a problem for problem which does not matching with the actual definition of derivation. The second student had also misconception that derivation is process of making rule without knowing the logic behind the rule. The above quotations clearly state that most of the students feel difficulty in gaining actual concept of derivation with symmetric mathematical logic.

Mixing of Concepts in Derivation

In relation to this student 5 uttered that "Honestly speaking I don't get what derivation talks about. I learned in this course that we find slope of and we find the points in graph." Another misconception hold by student-teachers regarding derivation is the process of finding a slope he has little knowledge and incomplete understanding of derivation concept. That student had just prior developed idea that in derivation is process of finding slope. From the response it could be concluded that student explained the derivation as pre-determined idea which he/she heard from instructor and colleagues. The students were unaware of the concept of slope and the logic of using slope concept in derivation.

Confusion in Variable and Function

This is related with concept of Leibniz notation in derivation. Students' response were like Student 1 communicated that "This question doesn't give sense up to know I haven't seen such problem" Student 3 conveyed that "I am Confused. If g is function then its derivation will be what, I don't know when g is variable" Student 5 said "Here it is a function. Its derivation will be same whether it is a function or variable". Five out of six students have difficulty in understanding chain rule. From their response it could be said that students struggle while using Leibniz notation. Students find it difficult to differentiate the function and variable in a given question. Students often assume that derivation is same whether it is a function or variable.

Procedural Approach of Solving Problem

This is associated with chain rule in derivation. Students gave these responses Student 1 and 2 stated "Yes I can find the derivation of it but I don't know about the rule. Five students out of six solves the question of chain rule. Surprisingly only one student was aware of rule used in problem rests of the students had memorized the steps of problems. From Students' responses it is concluded that students do not have the conceptual understanding when they solve problem involving chain rule is applied. They feel it difficult to connect the process of solving problem with its specific rule.

Difficulties in Understanding the Concept of Limit

Anthony Coggins (2014) assumes the importance of derivation and clearly states that limits are used to examine the function behaviour around points. Without learning limits, it is difficult to understand the rate of change. It is seen that limit process is inherent in derivation learners who struggle in limit, face difficulties in derivation .Therefore it is impossible to learn derivation without having concept of limit. This theme specifically talks about the common occurring difficulties in limit. It addressed second research question. More specifically, it addresses the underlying difficulties which restricts in learning derivation.

Difficulty in Gaining Initial Concept

Student 2 stated "I feel difficulty in limit. I studied it in intermediate, at that time I couldn't understand limit If we see the concept in language than it give a sense of boundary while in mathematics teacher tells us that it limit is thing which increase from both sides. It is going on and reaching to some point" .Three out of six students said that they feel difficulty in gaining initial concept of limit because they have studied limit many years ago which the barrier to grasp the concept was. So, it can be concluded that student-teachers were facing difficulty in understanding the basic concept of limit.

Difficulty in Making Real Life Connection

 Students 5 stated "When I stared learning limit at that time I was not getting that why are we learning limit. Teacher just tell us find limit of particular question. They don't ttell us real life application of limit or do not tell us why we study limit. "One out of six students acknowledged that it is difficult to connect the concept of limit with daily life. While learning limit he/she cannot understand where concept of limit is applied in real life situation and how it solves the complex real life problems. It seems that the limit concept requires students to do paper pencil work by following particular steps of problems without knowing the actual concrete logic behind the problem

Problems Involving Log and Radical

Students 1 said "Yes when there are problems involved radicals then I feel difficulty". Moreover, another student 5 also shared that "I become confused and perplexed when I solve problems in which radical is used". Two of the six students acknowledged they were unable to solve problems which contain log and radical whenever they came across the problems of limit which involve log. It can be said that student-teachers face difficulties in solving problems related to "log" in the calculus course.

Difficulties in Solving Word Problems of Derivation

Word problem are understood as important aspect of mathematical education which develop students' mental skills and logical thinking. The word problems boost creativity which enable learner to look for solution based on the given information. In a study Tall (1992) examined that in calculus emphasis is given symbolic manipulation rather than problem solving. As a result student feel difficulty in transforming word problems into calculus formulation. This theme particularly discusses the difficulties faced by students in solving word problem of derivation and addresses the third research question.

Language Difficulty

It is related with language of the word problem Student 2 said "The issue is with me that I can't understand language of word problem that what it actually conveys. Problems related with geometry, thermos and rules of circle. Similarly Student 3 uttered "The understanding of word problem depends upon its language. I can't understand language. Sometime we get supposition in fraction which is also difficult." Two out of four students' acknowledged that they feel difficulty in language. The above quotations reveals that the language used in word problem is major issue in solving word

problem. Learners find it difficult to get meaning of words used in problem and students also find it difficult to convert the word problem into mathematical formulation.

Difficulty in Extracting Information from Deriving

This deals with the information deriving difficulty in word problem. Student 5 said "When we are sorting data from word problem then we are given more than to values of single variable I get confused which value I have to take to solve problem". One of the six students talked about information deriving difficulty. The student's response shows that students often find it difficult to derive the important information from word problems which can be used to solve the problem.

Difficulty in Technique Selection in Word problems

The student teachers face difficulty in proper technique selection while solving the sums involving word problem. Student 5 told that "I feel difficulty. When I come across such problems I can't get which method will be used in problem. I do not know technique". One out of six students talked about technique selection difficulty. From the above response it is concluded that students also struggle in selecting the appropriate technique for solving word problem. Student often confuse which technique will be used in solving world problem.

Discussion on Findings

The findings of study revealed that student-teachers had many content related difficulties in the calculus course. More specifically, in both differential and integral calculus. Student-teachers who were studying in B.Ed. programme stated that they faced difficulties in getting the concept of Derivation, comprehending the concept of limit, and solving word problems of Derivation. Moreover, the difficulties faced by student-teachers in getting the concept of Derivation includes misconception about Derivation,

mixing of concepts in derivation, confusion in variables and functions , and procedural approach of problem solving. Whereas, difficulties in understanding the concept of limit includes gaining the initial concept of limit, making real life use of limit, and problems involves log and radical. In addition, the learning difficulties of student-teachers in word problem includes unclear mathematical language, extraction information while deriving information from word problems, difficulty in selecting the appropriate technique for solving word problems.

The findings of current study correlates with the studies conducted by Luneta & Makonye (2010) and Tarmizi (2010) because student-teachers perform low in calculus due to learning difficulties faced by them. These difficulties involved solving of equations, handling of operation having number and indices. The findings also relates with Tarmizi (2010) who found that conceptual misunderstanding of function and limit creates difficulty in learning of calculus i.e. differentiation, continuity and anti-derivatives. The studies highlighted that build the concept image is quite helpful in understanding the complex concepts of calculus in mathematics (Maharaj, 2013). However, understanding of concepts helps students to apply rules clearly.(Siyepu, 2015) suggested that whenever students misunderstood the concepts teachers ought to clear them first before teaching them calculus. Another reason of students were facing difficulties due to lack of understanding the basics of the calculus subject.

Conclusion

To conclude that students were facing difficulties in learning the concept of Derivation includes misconception about Derivation, mixing of concepts in derivation, confusion in variables and functions, and procedural approach of problem solving. Also, student-teachers were facing difficulties in comprehending the concept of limit, making real life use of limit, and problems involves log and radical. Thus, it can be concluded that

student-teachers learning difficulties in derivation can be addressed by introducing basic concepts first and they must be taught through conceptual understanding of the derivation in mathematics. Moreover, it is concluded that the learning of derivation requires the understanding of the rules, procedures as well as conceptual understanding of the problem. Hence, multiple approaches can be used to teach calculus and particularly derivation.

Recommendations

To address the content related difficulties of students in derivation. It is suggested that considerable attention should be given to explanations and illustrations regarding the meaning of limit and continuous function. The study suggests that instructors should first teach the basic concepts of positive, and negative limit so that students understand the concept. Teachers should teach important rules of log and surds so that students can solve the problems easily. The elementary concepts such as function, slope, and concept of tangent line should be taught before derivation so that students can understand the concept clearly. Rote learning of procedures and symbolic manipulation must be discouraged and conceptual understanding must be introduced while teaching derivation and chain rule. It is also suggested that teacher should teach word problems conceptually rather than memorizing the steps for solving the word problem. This study also suggests that Polya's approach of problem solving should be used while teaching word problems.

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